HEAT EXCHANGE IN VAPOR CONDENSATION ON A COLD LIQUID JET

V. I. Vishnyakov, K. V. Dement'eva, and A. M. Makarov

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Vapor condensation on a turbulent cold liquid jet is considered within the framework of the "new" Prandtl–Hertler mixing length theory. Our data on the heat transfer coefficient are in good agreement with the experimental results.

The problem of vapor condensation on a flat turbulent jet of a cold liquid was solved by G. N. Abramovich and A. P. Proskuryakov within the framework of the "old" Prandtl mixing length theory in the absence of the horizontal component of vapor velocity [1]. However, the results obtained in [1] differ from the experimental data by a factor of 1.5-2 [2]. In view of the pressing need for investigating the physical phenomena at the surface of a jet and the high intensity of the heat exchange and mass transport processes, the authors have considered a similar problem within the framework of the "new" Prandtl–Hertler mixing length theory.

Assume that a cold liquid with the assigned thermophysical parameters occupies the lower half-space and moves horizontally at the velocity u_l. As a result of condensation, a turbulent mixing layer develops at the interface between the liquid and the vapor space. It is assumed that complete instantaneous condensation of the flowing vapor occurs at the upper boundary of the mixing layer, that the phase transition surface is a plane, and that there are no vapor bubbles beyond the condensation boundary. The vapor velocity is v_V; it is caused only by the condensation process and is perpendicular to the surface of phase transition. The thermophysical parameters of the vapor are assigned. The coordinate system is positioned so that the axis of abscissas (longitudinal direction of the boundary layer) lies in the condensation plane (Fig. 1). According to the "new" Prandtl–Hertler mixing length theory, the equation of motion is [1]

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\varepsilon(x)\frac{\partial^2 u}{\partial y^2}, \tag{1}$$

where $\varepsilon = \varkappa b(u_1-u_0)$ is the coefficient of turbulent kinematic viscosity.

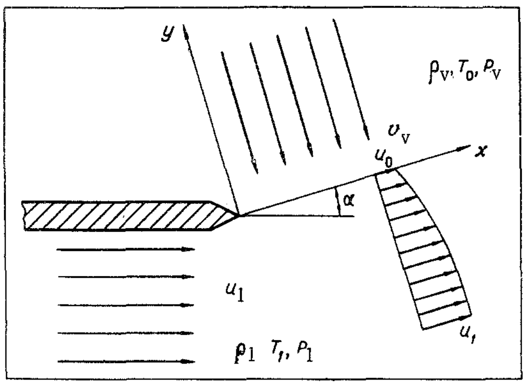

Fig. 1. Boundary layer of the jet.

We introduce the following notation:

$$\xi=\sigma y/x;\ u/U=\sigma F'(\xi);\ b=kx;\ U=u_1\cos\alpha;$$
$$\psi=\int u\,dy=xUF(\xi);\ v/U=\xi F'(\xi)-F(\xi). \tag{2}$$

After performing the substitution of variables in (1) by means of (2), we arrive at the self-similar equation of motion

$$F'''+2\sigma FF''=0, \tag{3}$$

where $\sigma = 1/2\sqrt{\varkappa k\lambda}$, and $\lambda = u_0/u_1$ is the ratio of flow velocities at the edges of the mixing zone at the jet section in question. We seek the solution of Eq. (3) in the form of a series with respect to integer powers of λ:

$$\sigma F=\sum_{n=0}^{\infty}\lambda^n F_n(\xi). \tag{4}$$

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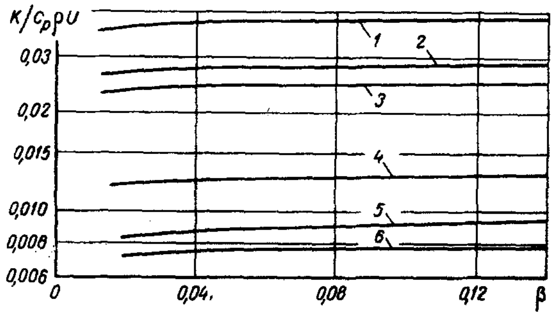

Fig. 2. Heat transfer coefficient as a function of the parameter β. 1) $\sigma = 10$, $\delta = 1.5$; 2) 14 and 1.5, respectively; 3) 16, 1.5; 4) 10, 2.5; 5) 14, 2.5; 6) 16, 2.5.

Retaining three terms of the expansion and assuming that $\sigma F_0(\xi)$, we obtain the following system of equations:

$$\begin{cases} F_1''' + 2\xi F_1'' = 0, \\ F_2''' + 2\xi F_2'' + 2F_1''F_1 = 0. \end{cases} \tag{5}$$

The formal solution of system (5) in the first approximation is given by

$$F_1(\xi) = F_1(0) + F_1'(0)\,\xi + F_1''(0)\int_0^{\xi} dz \int_0^{z} \exp(-x^2)\,dx\,, \tag{6}$$

where $F_1(0)$, $F_1'(0)$ and $F_1''(0)$ are the constants to be determined. The position of the lower boundary of the mixing layer is determined by setting the turbulent shearing stresses equal to zero, which is equivalent to the requirement that $F_1''(\xi)$ vanish. It is readily seen from [6] that these conditions are satisfied if $\xi \to -\infty$.

The longitudinal velocity of the mixing layer at its boundary with the basic liquid flow is equal to the projection of the basic flow velocity on the axis of abscissas:

$$\lim_{\xi \to -\infty} \sigma F'(\xi) = 1\,, \tag{7}$$

while, at the condensation surface, we have by definition

$$\sigma F'(0) = \lambda\,. \tag{8}$$

In order to satisfy conditions (7) and (8), we put

$$F_1'(0) = \frac{\lambda - 1}{\lambda} \text{ and } F_1''(0) = -\frac{2}{\sqrt{\pi}}\,\frac{1-\lambda}{\lambda}\,. \tag{9}$$

For determining $F_1(0)$, we use the continuity equation for the density of the momentum flow at the condensation boundary, which is written in the following form in projection on the x axis:

$$uv = -\overline{u'v'}\,. \tag{10}$$

We write the general expression for turbulent friction

$$-\overline{u'v'} = \frac{\tau}{\rho} = \varkappa b\,(u_1 - u_0)\left(\frac{\partial u}{\partial y}\right)_{y=0},$$

and, omitting the intermediate calculations, obtain the following:

$$-\sigma F'(0)\,\sigma F(0) = \frac{1}{2}\,\sigma F''(0)\,. \tag{11}$$

From relationships (11) and (9), we have

$$F_1(0) = \frac{1-\lambda}{\sqrt{\pi}\,\lambda^2}\,. \tag{12}$$

If we know the form of the function $F_1(\xi)$, we can determine the position of the phase transition boundary. Since

$$\operatorname{tg}\alpha = -\frac{v}{u} = \lim_{\xi \to -\infty}[F(\xi) - \xi F'(\xi)]\,, \tag{13}$$

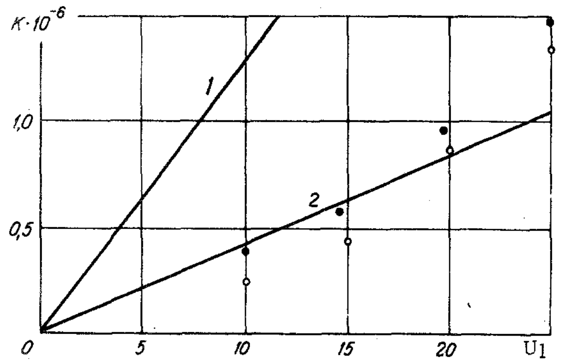

Fig. 3. Heat transfer coefficient (kcal/m^2·h·deg C) as a function of the characteristic flow velocity (m/sec). 1) Results borrowed from [1]; 2) our results (σ = 10, δ = 1.5, β = 1.0). The experimental data [2] for a jet flowing out of a 15-mm end-piece are shown by black points, while the data for a 10-mm end-piece are shown by white points.

we take into account (9) and (12) and obtain

$$\mathrm{tg}\,\alpha = \frac{1-\lambda^2}{\sigma\sqrt{\pi}\,\lambda}. \tag{14}$$

The equation closing the system of boundary conditions is the continuity equation for the energy flow density at the condensation surface:

$$-\rho_1 v(0)(i_v - i_1) = c_p \rho_1 \varepsilon_t \frac{\partial T}{\partial y}, \tag{15}$$

where $\varepsilon_t = \varkappa_t b_t(u_1 - u_0)$ is the empirical constant of the thermal boundary layer theory, $b_t = k_t x$ is the width of the thermal boundary layer, and k_t is the proportionality factor.

We introduce the dimensionless quantities

$$\beta = \frac{c_p(T_0 - T_1)}{i_v - i_1} \text{ and } (\delta\xi) = \frac{T_0 - T}{(T_0 - T_1)\sigma_t}, \tag{16}$$

where $\sigma_t = \delta\sigma$, and T_0 is the temperature at the condensation boundary.

With an allowance for (16), Eq. (15) can be written thus:

$$-\sigma F(0) = \frac{1}{2}\beta\sigma\theta'(0), \tag{17}$$

where $\sigma = 1/\delta\sqrt{2\varkappa_t k_t(1-\lambda)}$. Taking into account (12), we obtain from (17) an expression for β:

$$\beta = -\frac{2\delta(1-\lambda)}{\lambda\sqrt{\pi}\,\sigma\theta'(0)}. \tag{18}$$

The unknown value of the function $\theta'(0)$ figuring in expression (18) is found by solving the thermal problem. The equation of the thermal boundary layer

$$u\frac{\partial T}{\partial y} + v\frac{\partial T}{\partial y} = \varepsilon_T \frac{\partial^2 T}{\partial y^2}$$

is reduced to the following form by taking into account (15) and (16):

$$\theta''(\xi) + 2\delta^2[\sigma F(\xi)]\theta'(\xi) = 0. \tag{19}$$

According to (16), the boundary conditions are given by

$$T = T_0 \Rightarrow \theta(0) = 0, \qquad T = T_1 \Rightarrow \theta(-\infty) = \frac{1}{\sigma\delta}. \tag{20}$$

By solving Eq. (19) for the boundary conditions (20), we obtain

$$\theta'(0) = -\left[\sigma\delta\int_{-\infty}^{0}\exp\left(-\int_0^y 2\delta^2\sigma F(\xi)\,d\xi\right)dy\right]^{-1}. \tag{21}$$

By substituting (21) in (18), we obtain

$$\beta=\frac{2\delta^2(1-\lambda)}{\lambda\sqrt{\pi}}\int\limits_{-\infty}^{0}\exp\left(-\int\limits_{0}^{y}2\delta^2\sigma F(\xi)\,d\xi\right)dy. \tag{22}$$

The heat transfer coefficient in vapor condensation at the surface of the mixing layer is determined by

$$K=\frac{Q}{T_0-T_1}=\frac{c_p\rho_l\,\varepsilon_t}{T_0-T_1}\left(\frac{\partial T}{\partial y}\right)_{y=0}=\frac{c_p\rho_l U(1-\lambda)}{\lambda\sqrt{\pi}\,\sigma\beta}. \tag{23}$$

By substituting (22) in (23), we find

$$K=\frac{c_p\rho_l U}{\sigma 2\delta^2\int\limits_{-\infty}^{0}\exp\left(-\int\limits_{0}^{y}2\delta^2\sigma F(\xi)\,d\xi\right)dy}. \tag{24}$$

The dependence of the heat transfer coefficient K on σ, δ, and the parameter β is given in Fig. 2.

In Fig. 3, the experimental data obtained by N. M. Zinger [2] are compared with the calculations based on (24). This figure also shows the theoretical results obtained in [1] (curve 1). It is evident that our results show a better agreement with experimental data than the results obtained in [1]. This was to be expected, since the "new" Prandtl–Hertler mixing length theory has much greater potentialities because of the presence of two empirical constants (σ and δ), the first of which determines the thickness of the dynamic boundary layer, while the second determines the thickness of the thermal boundary layer.

NOTATION

u_l and v_v	are the velocities of the basic liquid flow and the vapor, respectively;
σ	is the first empirical constant of the theory;
u and v	are the mean velocity components;
ε	is the coefficient of turbulent kinematic viscosity;
ψ	is the stream function;
b	is the boundary layer width;
k	is the proportionality factor;
α	is the angle between the condensation plane and the direction of the basic liquid flow;
$\varkappa$	is the second empirical constant of the theory;
λ	is the ratio of flow velocities at the edges of the mixing zone at a given jet section;
v(0)	is the liquid velocity at the condensation surface;
$\varkappa_t$	is the empirical constant of the thermal boundary layer theory;
ρ_l	is the liquid density;
c_p	is the specific heat of the liquid;
i_v and i_l	are the enthalpies of the vapor and the liquid, respectively;
δ	is the relative width of the thermal boundary layer;
T_0	is the temperature at the condensation boundary;
T_1	is the temperature of the main body of liquid flow;
T	is the present temperature;
K	is the heat transfer coefficient;
Q	is the thermal flux from the vapor space.

LITERATURE CITED

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2. N. M. Zinger, in: Heat Exchange during Variation of the State of Aggregation of Matter [in Russian], GÉI (State Power Engineering Press) (1953).